Int. J. Heat Mass Transfer. Vol. 20, pp. 753–761. Pergamon Press 1977. Printed in Great Britain

TRANSIENT RESPONSE OF PERIODIC-FLOW REGENERATORS

A. J. WILLMOTT and A. BURNS
Department of Computer Science, University of York, Heslington, York, YO1 5DD, U.K.

(*Received* 29 *June* 1976 *and in revised form* 5 *October* 1976)

Abstract—The heat storing packing, an intrinsic part of a thermal regenerator, imposes an inertia which prevents instantaneous response to changes in operating conditions. Presented here in non-dimensional graphical form are the responses of a thermal regenerator, initially at cyclic equilibrium, to permanent step changes in inlet gas temperature and in gas flow rate. It is demonstrated that reduced length Λ parameterises this thermal inertia in the balanced case. The effect of imbalance in regenerator operation upon thermal inertia is investigated.

NOMENCLATURE

- A, regenerator heating surface area [m^2];
- C, specific heat of storing matrix [J/kg K];
- $\bar{h}$, bulk heat-transfer coefficient [W/m^2 K];
- L, length of regenerator [m];
- M, mass of heat storing matrix [kg];
- m, mass of gas resident in regenerator [kg];
- P, length of operating period [s];
- p, ratio of reduced lengths, Λ'/Λ'';
- S, specific heat of gas [J/kg K];
- T, temperature of heat storing matrix [K];
- t, temperature of gas [K];
- W, flow rate of gas [kg/s];
- y, distance from regenerator entrance [m].

Greek symbols

- β, degree of unbalance;
- γ, ratio of the magnitude of a step change to that of the threshold value;
- ξ, dimensionless length;
- $\varepsilon f_1, \varepsilon f_2$, dimensionless measure of transient response—rotary regenerator;
- $\varepsilon g_1, \varepsilon g_2$, dimensionless measure of transient response—fixed bed regenerator;
- η, dimensionless time;
- η_{REG}, thermal ratio;
- Θ, total dimensionless time required to regain equilibrium following a step change in operating conditions;
- θ, time [s];
- Λ, reduced length $\bar{h}A/WS$ [dimensionless];
- Π, reduced period $\bar{h}A(P-m/W)/MC$ [dimensionless].

Subscripts

- IN, inlet;
- OUT, outlet;
- m, chronological mean;
- H, harmonic mean.

Superscripts

- $'$, refers to hot period;
- $''$, refers to cold period;
- (n), refers to cycle number n;
- (θ), refers to time θ;
- (0), refers to cyclic equilibrium prior to step change;
- (∞), refers to cyclic equilibrium after step change.

INTRODUCTION

MOST previous theoretical work confines itself to the periodic behaviour of thermal regenerators, and this is acceptable when the regenerator is required to work under rarely varying operating conditions. However, for some regenerators, the effect of thermal inertia when changes in operation take place becomes important. Cowper stoves are required to meet a chronologically varying thermal demand from the ironmaking furnace. In this case the thermal inertia has a "nuisance value" and control strategies must be developed if these varying thermal loads are to be accommodated with minimal fuel consumption. Air conditioning regenerators are required to operate in varying climatic conditions; here the thermal inertia is exploited to smooth out the effect of these variations. It follows that an understanding of the thermal inertia of regenerative systems is fundamental in all such areas of application.

London *et al.* [1] examined the transient response of the exit gas temperatures arising from a step input change in inlet gas temperature. They were concerned with gas turbine plant thermal regenerators. They restricted their considerations to balanced regenerators ($W'S'P' = W''S''P''$) and showed that for $\Lambda'/\Pi' = \Lambda''/\Pi'' > 100$, the transient response before cyclic equilibrium is re-established, does not depend upon reduced period and can be expressed solely as a function of a reduced length. In their paper, they extended earlier work by London *et al.* [2] and by Cima and London [3] who dealt with the recuperative heat exchanger. Green [4] found, similarly, that for symmetric regenerators ($\Lambda' = \Lambda''$, $\Pi' = \Pi''$), the total dimensionless time to re-establish cyclic equilibrium following a permanent step change in inlet gas temperature can be approximated by a function of reduced length Λ only. Green proceeded also to investigate the effect of step changes in gas flow rate and showed that the time to cyclic equilibrium to be reattained is a non-linear function of the magnitude of the step change; for the particular Cowper stove configuration he considered, this time reached a maximum value of about 2–3 weeks for step changes of the order of 10% in gas flow rate (hot period).

The implication of Green's work is that except where operating conditions rarely change, it is unlikely that Cowper stoves have an opportunity to work at cyclic equilibrium. The object of this paper is to enhance our understanding of the transient (non-periodic) behaviour of thermal regenerators for both changes in flow rate and inlet gas temperature. In particular the effect of imbalance ($W'S'P' \neq W''S''P''$) in regenerator operation upon transient behaviour is investigated.

MATHEMATICAL MODEL

The temperature behaviour of a thermal regenerator is represented by the differential equations (1) and (2) discussed by Hausen [5].

$$\frac{\partial t}{\partial \xi} = T - t \tag{1}$$

$$\frac{\partial T}{\partial \eta} = t - T \tag{2}$$

where ξ and η are dimensionless measures of distance and time respectively.

$$\xi = \frac{\bar{h}Ay}{WSL} \tag{3}$$

$$\eta = \frac{\bar{h}A}{MC}\left(\theta - \frac{my}{WL}\right). \tag{4}$$

Hausen proposed the dimensionless parameters "reduced length" Λ and "reduced period" Π to characterise each period of operation, where

$$\Lambda = \frac{\bar{h}A}{WS} \tag{5}$$

$$\Pi = \frac{\bar{h}A}{MC}\left(P - \frac{m}{W}\right). \tag{6}$$

The superscripts $'$ and $''$ are used to distinguish hot and cold periods. The assumptions embodied in this mathematical idealisation are discussed by Willmott and Thomas [6]. The following boundary conditions appertain:

(i) The hot and cold period inlet gas temperatures t'_{in} and t''_{in} remain constant during any cycle. Step changes are made at the beginning of the period under consideration but the operating conditions then remained unchanged until cyclic equilibrium is re-established.

(ii) The regenerator is operated in contra-flow mode. All temperatures are measured from the gas entrance in each period and thus the solid temperature profile at the beginning of any period is related to the profile at the end of the preceding period by the equations (7) and (8)

$$T'(0, y) = T''(P'', L-y) \tag{7}$$

$$T''(0, y) = T'(P', L-y) \tag{8}$$

for $0 \leqslant y \leqslant L$.

SOLUTION OF DIFFERENTIAL EQUATIONS

The finite difference method of solution adopted in this paper is that of Willmott [7] with an improved convergence criteria (see Appendix). London *et al.* [1] produced their graphical solutions using both an analogue computer and the numerical methods of Lambertson [8] developed for digital computation.

RESPONSE TO A STEP CHANGE IN OPERATING CONDITIONS

London *et al.* [1] illustrated the responses to changes in operating conditions for a rotary regenerator by defining the following dimensionless parameters $\varepsilon f_1^{(\theta)}$ and $\varepsilon f_2^{(\theta)}$, given

$$\varepsilon f_1^{(\theta)} = \frac{t'_{OUT}{}^{(\theta)} - t'_{OUT}{}^{(\theta=0)}}{t'_{OUT}{}^{(\theta=\infty)} - t'_{OUT}{}^{(\theta=0)}} \tag{9}$$

$$\varepsilon f_2^{(\theta)} = \frac{t''_{OUT}{}^{(\theta)} - t''_{OUT}{}^{(\theta=0)}}{t''_{OUT}{}^{(\theta=\infty)} - t''_{OUT}{}^{(\theta=0)}} \tag{10}$$

where $0 \leqslant \theta \leqslant \infty$.

At equilibrium, immediately prior to a step change ($\theta = 0$), $\varepsilon f_1^{(0)} = \varepsilon f_2^{(0)} = 0$; once cyclic equilibrium has been restored ($\theta = \infty$) $\varepsilon f_1^{(\infty)} = \varepsilon f_2^{(\infty)} = 1$.

London restricted his consideration to rotary regenerators where the exit gas temperature is an average over the whole face of the rotating heat storing mass (chequerwork) currently exposed to the gas stream under consideration. Since this type of regenerator operates continuously London was able to express εf_1 and εf_2 as continuous functions of time.

This approach ignores two factors namely: (1) the thermal state of each sector of the chequerwork will be different at the moment when a step change is effected and (2) the response on the cold side to changes on the hot side (and vice versa) will be delayed by a time determined by the speed of rotation of the chequerwork. London was able to ignore these problems by considering cases where $\Lambda'/\Pi' = \Lambda''/\Pi'' > 100$, that is where the angular velocity of the rotor is great relative to the gas flow rate.

In our work we assume that for fixed bed regenerators, step changes are made at the beginning of the period under consideration and the responses εg_1 and εg_2 are measured in terms of the chronological average exit gas temperatures, $t'_{OUT,m}{}^{(n)}$ and $t''_{OUT,m}{}^{(n)}$ for the nth cycle following the step change. We denote

$$\varepsilon g_1{}^{(n)} = \frac{t'_{OUT,m}{}^{(n)} - t'_{OUT,m}{}^{(0)}}{t'_{OUT,m}{}^{(\infty)} - t'_{OUT,m}{}^{(0)}} \tag{11}$$

$$\varepsilon g_2{}^{(n)} = \frac{t''_{OUT,m}{}^{(n)} - t''_{OUT,m}{}^{(0)}}{t''_{OUT,m}{}^{(\infty)} - t''_{OUT,m}{}^{(0)}} \quad (12)$$

$t'_{OUT,m}{}^{(0)}$ and $t''_{OUT,m}{}^{(0)}$ denote the mean exit gas temperatures at equilibrium in the cycle just prior to the step change; $t'_{OUT,m}{}^{(\infty)}$ and $t''_{OUT,m}{}^{(\infty)}$ are the corresponding temperatures once cyclic equilibrium has been attained again.

$\varepsilon g_1{}^{(n)}$ represents the exit gas temperature response on the *same side* as that on which the step change in inlet gas temperature or gas flow rate is made; $\varepsilon g_2{}^{(n)}$ represents the response on the *opposite side* to that of the step change. The linear nature of the model allows us in this work, to consider step changes made on the hot side alone. The results are equally applicable to step changes on the cold side if $\varepsilon g_1{}^{(n)}$ [see equations (11) and (12)] is interpreted as $\varepsilon g_2{}^{(n)}$ and vice versa.

Our work is applicable to rotary regenerators if it can be assumed that the different thermal states of the sectors of the rotor at the moment of the step change can be ignored. For this case, $\varepsilon g_1{}^{(n)}$ and $\varepsilon g_2{}^{(n)}$ correspond to $\varepsilon f_1[\theta = n(P'+P'')-P'']$ and $\varepsilon f_2[\theta = n(P'+P'')]$ for step changes made on the hot side. The delay in response on the cold side to such step changes on the hot side is thus represented.

The overall performance at both the new and old equilibrium conditions is defined by the thermal ratios η'_{REG} and η''_{REG}. For changes in inlet gas temperature we have

$$\eta'_{REG} = \underset{\text{New equilibrium}}{\frac{t'^{*}_{IN} - t'_{OUT,m}{}^{(\infty)}}{t'^{*}_{IN} - t''_{IN}}} = \underset{\text{Old equilibrium}}{\frac{t'_{IN} - t'_{OUT,m}{}^{(0)}}{t'_{IN} - t''_{IN}}} \quad (13)$$

$$\eta''_{REG} = \underset{\text{New equilibrium}}{\frac{t''_{OUT,m}{}^{(\infty)} - t''_{IN}}{t'^{*}_{IN} - t''_{IN}}} = \underset{\text{Old equilibrium}}{\frac{t''_{OUT,m}{}^{(0)} - t''_{IN}}{t'_{IN} - t''_{IN}}} \quad (14)$$

where t'^{*}_{IN} is the inlet gas temperature after the step change.

When a step change is made in the gas inlet temperature, Λ', Λ'', Π' and Π'' are unaffected and thus the thermal ratios at the new equilibrium are equal to those at the equilibrium in force prior to the step change (13) and (14). This enables $t'_{OUT,m}{}^{(\infty)}$ and $t''_{OUT,m}{}^{(\infty)}$ to be calculated before the step change and $\varepsilon g_1{}^{(n)}$ and $\varepsilon g_2{}^{(n)}$ to be evaluated from $t'_{OUT,m}{}^{(n)}$ and $t''_{OUT,m}{}^{(n)}$ as the regenerator simulation proceeds.

However, when step changes are made in gas flow rate, some or all of the parameters Λ', Λ'', Π' and Π'' are altered. Unless the thermal ratios for periodic behaviour for the revised dimensionless parameters are known, $\varepsilon g_1{}^{(n)}$ and $\varepsilon g_2{}^{(n)}$ can only be calculated once cyclic equilibrium is re-established and $t'_{OUT,m}{}^{(\infty)}$ and $t''_{OUT,m}{}^{(\infty)}$ have been computed.

London [1] exhibited the response of a regenerator to changes in operation by displaying graphically the variation of εf_1 and εf_2 with time. Green [4] on the other hand compared the effect of different step changes in operation in terms of the total time taken to re-establish cyclic equilibrium. In this paper, we employ both approaches and investigate the dependence of εg_1 and εg_2 upon the parameters of the system which are reduced lengths and reduced periods for the hot and cold periods as they describe the regenerator both before and after the step change.

A further parameter is the magnitude of the step change although as London pointed out, the linear nature of the model implies that εg_1 and εg_2 will not be dependent upon the size of any step change made in inlet gas temperature. We will show that this is not the case for step changes in flow rate.

STEP CHANGE IN INLET GAS TEMPERATURE

Previous work here has been restricted to symmetric regenerators ($\Lambda = \Lambda' = \Lambda''$, $\Pi = \Pi' = \Pi''$). We discuss this case and then extend the analysis to include, unsymmetric-balanced ($\Lambda'/\Pi' = \Lambda''/\Pi''$) and unbalanced ($\Lambda'/\Pi' \neq \Lambda''/\Pi''$) regenerators.

Symmetric case

In Fig. 1 are displayed the responses εg_1 and εg_2 as functions of the dimensionless time parameter η for a reduced length $\Lambda = 10$. It will be seen that the response is independent of reduced period Π for $\Lambda/\Pi = 100$, 200 and 400, thereby confirming the observations of London *et al.* [1] for $\Lambda/\Pi > 100$. We have investigated cases outside the ranges examined by London and we have shown, see Fig. 2, that for example with $\Lambda = 28$, εg_1 and εg_2 do exhibit some dependence on reduced period Π for $\Pi = 16$, 8, 4 and 2. Nevertheless the total dimensionless time Θ needed to re-establish cyclic equilibrium remains independent of reduced period. The regenerator approaches cyclic equilibrium asymptotically. However this equilibrium is considered here to have been re-established when the measure of the convergence (see Appendix) falls below $\varepsilon = 10^{-4}$.

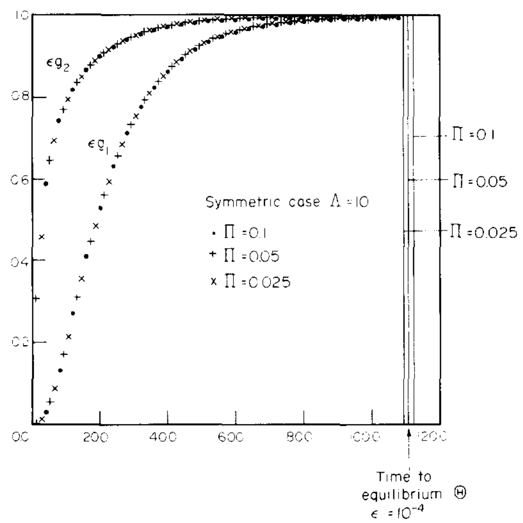

FIG. 1. Responses εg_1 and εg_2 to step changes in hot inlet gas temperature for $\Lambda/\Pi > 100$.

The response displayed in Figs. 1 and 2 together with London's results indicate the response εg_1 on the hot side to a step change on the hot side converges towards equilibrium more slowly than εg_2, the corresponding response on the cold side. Employing the

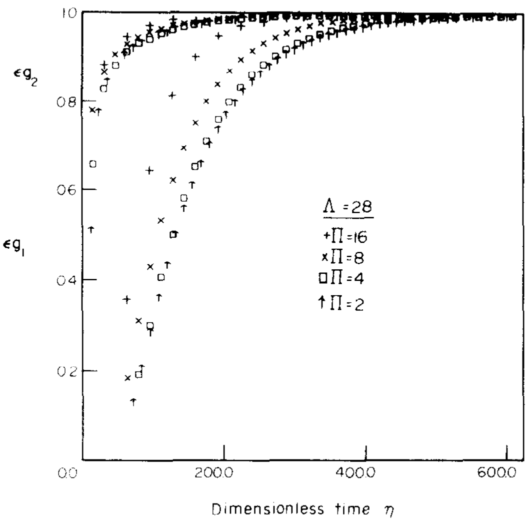

FIG. 2. Dependence of responses εg_1 and εg_2 upon Π for step changes in hot inlet gas temperature. (Symmetric case.)

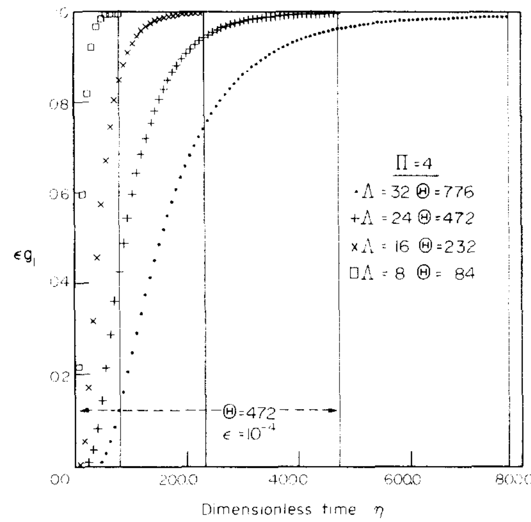

FIG. 3. The effect of reduced length Λ upon the response εg_1 and the time to equilibrium Θ. (Symmetric case.) Vertical lines denote time when cyclic equilibrium is restored.

criteria described in the Appendix, no difference can be detected between the dimensionless times required for εg_1 and εg_2 to converge. It follows that without loss of generality, it is possible to examine the response εg_1 alone to facilitate clearer graphical presentation.

Step changes in inlet gas temperature (or gas flow rate) on the hot side result in perturbations in the temperature of the solid packing; indeed the effect of the thermal capacity of the packing conceals the effect of the step change upon exit gas temperature in the hot period. However, these perturbations have an immediate effect upon the gas flowing through the regenerator in the opposite direction in subsequent cold periods. As a consequence the manifestation of the effect of step changes upon the exit gas temperatures appear to be much slower on the side on which the step change is imposed.

The "size" of the force imposing oscillations of temperature upon the solid packing can be measured by the thermal capacity flow rate WS of the gas, in either period of operation relative to the corresponding "size" $\bar{h}A$ of the regenerator. What we might expect is that the larger the *relative* size of WS, that is the smaller the reduced length $\Lambda = \bar{h}A/WS$, the sooner the packing will respond to changes in operation.

That the reduced length Λ is an effective measure of the inertia of a symmetric regenerator is illustrated in Fig. 3 where εg_1 and Θ are displayed for reduced period fixed at $\Pi = 4$ for different reduced lengths $\Lambda = 32$, 24, 16 and 8. The initial lag in the response, which increases with reduced length is typical of a distributed parameter system of which a thermal regenerator system is an example.

The revised convergence criteria (see Appendix) has been used to improve Green's formula relating the total dimensionless time to re-establish cyclic equilibrium Θ to reduced length Λ. Green proposed that for $\Lambda \leqslant 30$,

$$\Theta = 0.524\Lambda^2 + 3.760\Lambda + 9.432. \qquad (15)$$

We suggest the following equation is more accurate

$$\Theta = 0.622\Lambda^2 + 4.144\Lambda + 6.464 \qquad (16)$$

for $\Lambda \leqslant 40$. It is recognised that Θ is also a function of the convergence criterion. Equation (16) assumes $\varepsilon = 10^{-4}$ (see Appendix). For another value of $\varepsilon = E$ (e.g. 10^{-2}), we can define the time to equilibrium to be Θ_E which can be estimated using

$$\Theta_E = \Theta \log_e E/\log_e \varepsilon.$$

For example if $E = 10^{-2}$

$$\Theta_E = \Theta \log_e(10^{-2})/\log_e(10^{-4})$$

and $$\Theta_E = \Theta/2.$$

Equation (16) was obtained by fitting a quadratic function in Λ by the method of least squares to the values of Θ which we computed by regenerator simulation for different values of Λ. It was noted that Θ, the total reduced time to regain equilibrium computed in this way is always obtained from an integral number of cycles; the coefficients of the quadratic were therefore restrained in such a way that the time to equilibrium estimated using the quadratic never exceeded the corresponding time calculated using the simulation at any of the data points used.

Unsymmetric-balanced regenerators

Since $\Lambda'/\Pi' = \Lambda''/\Pi''$ for unsymmetric but balanced regenerators, the hot thermal ratio η'_{REG} is equal to the cold thermal ratio η''_{REG}. Hausen [9] proposed the use of "harmonic means", Λ_H and Π_H defined as:

$$\frac{1}{\Lambda_H} = \frac{1}{2}\left(\frac{1}{\Lambda'} + \frac{1}{\Lambda''}\right)$$

$$\frac{1}{\Pi_H} = \frac{1}{2}\left(\frac{1}{\Pi'} + \frac{1}{\Pi''}\right)$$

to parameterise the equilibrium condition of the unsymmetric-balanced regenerator in terms of a symmetric regenerator with $\Lambda = \Lambda_H$ and $\Pi = \Pi_H$. Iliffe

[10] first confirmed the acceptability of this parameterisation using harmonic means, for the calculation of thermal ratios. The degree of symmetry can be specified by a parameter p where $p = \Lambda'/\Lambda'' = \Pi'/\Pi''$ from which it follows that

$$\Lambda' = (p+1)\Lambda_H/2 \text{ and } \Pi' = (p+1)\Pi_H/2$$

and that $\Lambda'/\Pi' = \Lambda''/\Pi'' = \Lambda_H/\Pi_H$. Iliffe examined cases over the range $0 \leqslant \Pi'' \leqslant 24$ and $0 \leqslant \Lambda'' \leqslant 24$ for $p = 2$ and $0 \leqslant \Pi'' \leqslant 18$ and $0 \leqslant \Lambda'' \leqslant 18$ for $p = 3$.

We have verified that the transient behaviour of this type of regenerator can also be adequately parameterised by the use of harmonic means. The responses εg_1 and εg_2 to a step change on the hot side inlet gas temperature for the symmetric regenerator with $\Lambda = \Lambda_H = 24$ and $\Pi = \Pi_H = 8$ follow very closely the responses εg_1 and εg_2 respectively for the corresponding unsymmetric-balanced cases for:

(i) $\Lambda' = 36$, $\Lambda'' = 18$, $\Pi' = 12$ and $\Pi'' = 6$,

(ii) $\Lambda' = 18$, $\Lambda'' = 36$, $\Pi' = 6$ and $\Pi'' = 12$.

Note that because the step change considered is applied to the hot inlet gas temperature, it is necessary to consider both cases (i) and (ii) which correspond to $p = 2$ and $p = \frac{1}{2}$. Further the response εg_1 for $p = 2$ and that for $p = \frac{1}{2}$ lie on opposite sides of the response εg_1 for the corresponding symmetric case ($p = 1$).

The time Θ to equilibrium can be calculated approximately using the revised Green formula for the equivalent symmetric case

$$\Theta_H = 0.622\Lambda_H^2 + 4.144\Lambda_H + 6.464 \qquad (17)$$

for $\Lambda_H \leqslant 40$, $\varepsilon = 10^{-4}$. From Θ_H can be obtained the actual dimensionless time Θ for the unsymmetric-balanced regenerator to regain equilibrium.

$$\Theta = \frac{\Theta_H}{4}(1+p)\left(1+\frac{1}{p}\right).$$

The approximation (17) provides a relative accuracy of 10% or less.

Unbalanced regenerators

In the balanced case, the forces imposing the results of the step change in operation in the hot period might be regarded as being partly counter-balanced by opposing forces in the cold period. For example the propagation of the heat front down the length of the packing resulting from a step change in inlet gas temperature in the hot period will be delayed to a greater or lesser extent by the action of the cold gas passing through the packing in successive cold periods. When $\Lambda'/\Pi' \neq \Lambda''\Pi''$, then the two thermal ratios η'_{REG} and η''_{REG} are not equal and the regenerator is also unbalanced in the sense that the forces imposing temperature changes in the packing in the hot period are not matched by the counter-forces operating in the cold period. When this happens, the packing responds more quickly to the changes imposed upon it. An extra parameter $\beta = (\Lambda'/\Pi')/(\Lambda''/\Pi'')$ is needed to account for the effect of the unbalance in the system upon the transient performance. The use of harmonic mean reduced length Λ_H is extended to this case. The

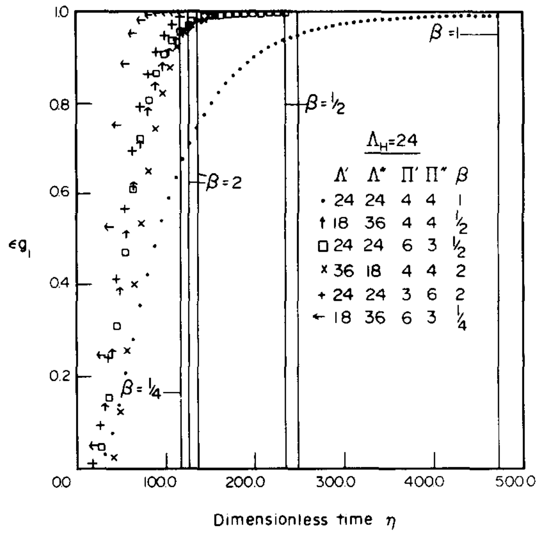

FIG. 4. Response εg_1 to step changes in hot inlet gas temperature: unbalanced regenerators ($\Lambda'/\Pi' \neq \Lambda''/\Pi''$). Vertical lines denote time when equilibrium is restored.

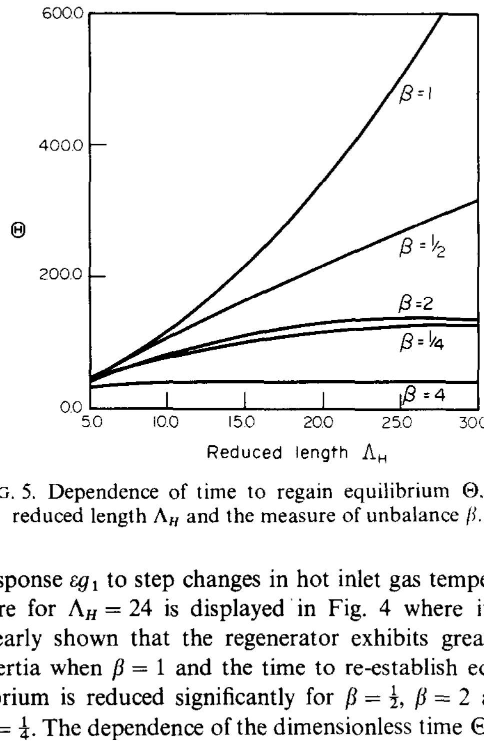

FIG. 5. Dependence of time to regain equilibrium Θ, on reduced length Λ_H and the measure of unbalance β.

response εg_1 to step changes in hot inlet gas temperature for $\Lambda_H = 24$ is displayed in Fig. 4 where it is clearly shown that the regenerator exhibits greatest inertia when $\beta = 1$ and the time to re-establish equilibrium is reduced significantly for $\beta = \frac{1}{2}$, $\beta = 2$ and $\beta = \frac{1}{4}$. The dependence of the dimensionless time Θ to regain equilibrium upon the parameters Λ_H and β is clearly displayed in Fig. 5.

STEP CHANGES IN GAS FLOW RATE

Green [4] considered the effect of step change on a regenerator with symmetric configuration *prior* to the step change. He observed that the total time required to re-establish equilibrium reached a maximum for changes of approximately 10–20% and then was reduced for larger step changes in flow rate.

We extend our results for step changes in inlet gas temperature to step changes in gas flow rate by considering the state of the regenerator (symmetric or unsymmetric, balanced or unbalanced) *after* the step change, because the final condition towards which the regenerator will "converge" is determined by this final state. It is shown that these results can be used to predict the transient responses that Green observed.

Equations (5) and (6) indicate the dependence of Λ and Π on W and $\bar{h}$. If we consider the case where $\bar{h}$ is approximately linearly proportional to W, Λ is independent of flow rate and the response of the regenerator following a step change in the hot period gas flow rate can be investigated by considering step changes in Π' alone.

When the step change in Π' is greater than the threshold value ($\gamma > 1$) the transient response of the regenerator is parameterised only by Λ_H and β. The threshold value is usually less than 50%; for $\Lambda = 10$ $\Pi = 4$ it is 10% and for $\Lambda = 20$ $\Pi = 4$ it is 35%. The dependence of the transient response upon reduced length, reduced period and β for step changes of 50%, well beyond the threshold limit is therefore investigated. Our results are presented in Figs. 8 and 9.

For such 50% changes to Π', for the symmetric case, εg_1 and Θ are observed in Fig. 8 to be independent

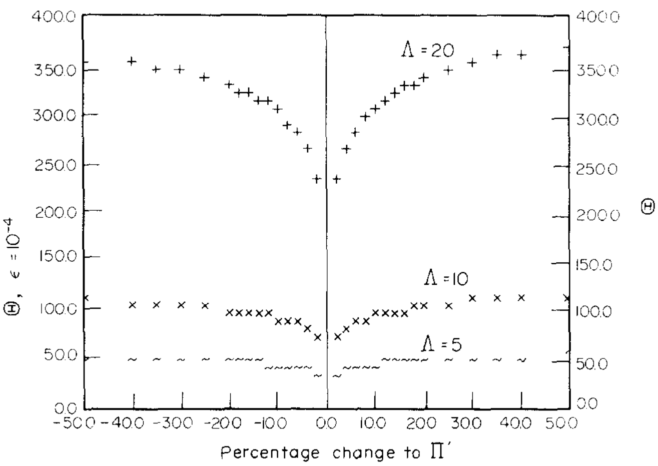

FIG. 6. Dependence of time to equilibrium Θ on reduced length Λ for varying degrees of percentage change of Π'. The final state of the regenerator is symmetric with reduced period $\Pi = 4$.

Up to a certain threshold, the time taken to re-establish equilibrium after a step change in hot gas flow rate, increases with the size of the step change. Beyond this threshold value of the percentage change made to the reduced period Π' and implicitly to the hot gas flow rate the transient response of the regenerator is dependent only upon the final operating conditions. The threshold value increases with reduced length. These features are displayed in Fig. 6 for the case where the final state of the regenerator is symmetric with $\Pi = 4$. Reduced lengths $\Lambda = 5$, 10 and 20 are considered. However, it will be shown the parameter γ

$$\gamma = \frac{\text{Magnitude of step change}}{\text{Magnitude of step change at threshold}}$$

can be used to relate the transient response below the threshold value to the response beyond it.

Indeed, the transient response of the regenerator to step changes in Π' (hot gas flow rate) can be parameterised by the harmonic mean reduced length Λ_H, and the unbalance factor β, as in the case of step changes in inlet gas temperature, together with the additional factor γ. The relatively small effect of γ upon the dimensionless time Θ to regain cyclic equilibrium in the case of balanced regenerators is displayed in Fig. 7 for $2 \leqslant \Lambda_H \leqslant 30$ and $\gamma = 1, \frac{1}{2}$ and $\frac{1}{4}$. From Figs. 6 and 7 it will be seen that the larger the reduced length, the more sensitive is the transient response to the value of γ.

of reduced period. The total reduced time to equilibrium can thus be expressed as a function of Λ only (as $\beta = 1$), equation (18), which is of similar type to equation (16) relating Θ to Λ for step changes in inlet gas temperature.

$$\Theta = 0.72\Lambda^2 + 4.2\Lambda + 8 \text{ for } \Lambda \leqslant 30, \varepsilon = 10^{-4}. \quad (18)$$

For unsymmetric regenerators, harmonic means again provide adequate parameterisation of the transient behaviour. Although small differences arise between the actual transient response of an unsym-

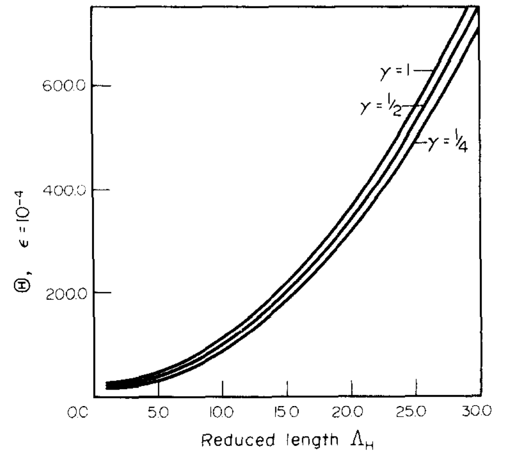

FIG. 7. Dependence of time to equilibrium Θ, upon reduced length Λ_H and γ the step size parameter.

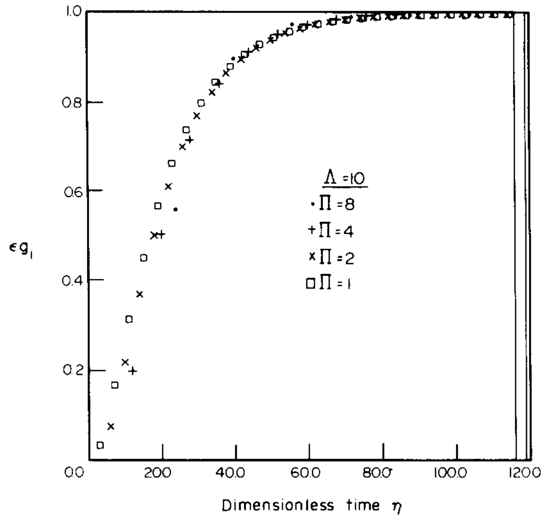

FIG. 8. Dependence of response εg_1 upon Π for step changes in Π'. (Symmetric case.)

metric-balanced regenerator and that of the corresponding equivalent symmetric regenerator, nevertheless the total time to re-establish equilibrium for $\varepsilon = 10^{-4}$ remains accurately represented by this equivalent symmetric case.

The effect of unbalance on the system, parameterised by β, is illustrated in Fig. 9 where the response εg_1 is displayed for Λ_H fixed at 10 and $\beta = 1, 2, \frac{1}{2}, 4$ and $\frac{1}{4}$. The greatest inertia is again exhibited by the balanced regenerator $\beta = 1$ (compare with Fig. 4 for step change in inlet gas temperature).

Green's results, can now be interpreted in terms of our observations. Step changes are made to a regenerator which is initially symmetric in Green's work. For small step changes, the regenerator's final state will still be approximately symmetric and the time to re-establish cyclic equilibrium will increase with the size of the step change γ. For larger step changes, the regenerator becomes increasingly unbalanced and the dimensionless time Θ to regain equilibrium decreases. There is therefore a value of the step change in gas flow rate for a particular regenerator configuration at which the time to equilibrium is a maximum beyond which the effect of the unbalance β becomes increasingly predominant.

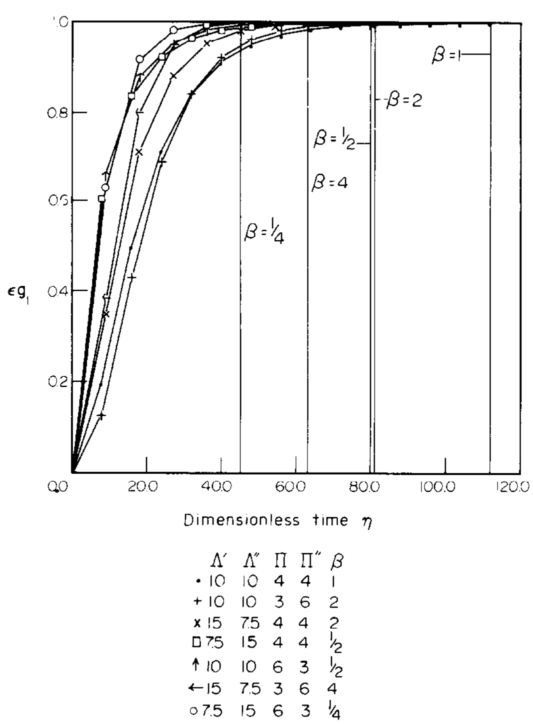

FIG. 9. Response εg_1 to a step change in Π' for unbalanced regenerators ($\Lambda'/\Pi' \neq \Lambda''/\Pi''$). Vertical lines denote time when equilibrium is restored.

CONCLUSIONS

The transient response of a thermal regenerator to step changes in either inlet gas temperature or gas flow rate can be characterised by the harmonic mean of the reduced lengths, Λ_H and degree of unbalance β. The total dimensionless time to equilibrium is independent of reduced period and can be made up of a few long cycles or many short cycles. The inertia of the regenerator increases with reduced length Λ_H; for a particular value of Λ_H, a maximum inertia is exhibited by a balanced regenerator, $\beta = 1$. Any unbalance ($\beta \neq 1$) reduces the time taken for the regenerator to attain cyclic equilibrium.

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APPENDIX

Criteria for Detecting Cyclic Equilibrium

Willmott [7] proposed a pseudo-thermal ratio for the nth cycle

$$Z'_n = \frac{t'_{IN} - t'_{OUT,m}{}^{(n)}}{t'_{IN} - t''_{IN}}$$

$$Z''_n = \frac{t''_{OUT,m}{}^{(n)} - t''_{IN}}{t'_{IN} - t''_{IN}}$$

and suggested that provided

$$|Z''_n - Z''_{n-1}| < \varepsilon \tag{A1}$$

where ε is a small number, say 10^{-4} or 10^{-5}, cyclic equilibrium could be considered to have been achieved in the nth cycle. This criterion is satisfactory for many regenerator calculations.

If the true equilibrium value of Z'' is α'', that is if

$$\lim_{n\to\infty} Z''_n = \alpha''$$

then the true convergence criterion is $|Z''-\alpha''| < \varepsilon$. The criterion (A1) overcomes the fact that α'' is unknown by presuming that if

$$|Z''_n - Z''_{n-1}| < \varepsilon$$

then $$|Z''_n - \alpha''| < \varepsilon.$$

However, if the regenerator possesses a large thermal inertia and convergence of Z''_n towards α'' is very slow, the simulation can still be some way from equilibrium, even when the values of Z''_n and Z''_{n-1} are close together; thus the criterion may give a misleading conclusion.

The asymptotic convergence of Z''_n (or Z'_n) can be approximated exponentially. It follows that an estimate of α''^* of the value α'' can be extrapolated [11] using Aitken's formula:

$$\alpha''^* = Z''_n - \frac{(Z''_n - Z''_{n-1})^2}{Z''_n - 2Z''_{n-1} + Z''_{n-2}}.$$

This assumes that the discrepancy $(Z''_n - \alpha'')$ decreases exponentially with n. Using this estimate the present authors have found an improved criteria, replacing the true criterion

$$|Z''_n - \alpha''| < \varepsilon$$

by the approximate criterion

$$|Z''_n - \alpha''^*| < \varepsilon$$

i.e.
$$\left|\frac{(Z''_n - Z''_{n-1})^2}{Z''_n - 2Z''_{n-1} + Z''_{n-2}}\right| < \varepsilon. \tag{A2}$$

The simulation for regenerators with large inertia can be safely regarded as having reached cyclic equilibrium when the criterion (A2) is satisfied.

The integration of the differential equations (1)–(2) over a complete cycle can be regarded as a function f acting upon Z so that

$$Z''_n = f(Z''_{n-1})$$

if this function and its derivative could be expressed explicitly, then the iterative process would converge to α'' provided a value of ξ existed on the range between Z''_{n-1} and α'' such that

$$|f'(\xi)| < 1.$$

However the function f cannot be expressed explicitly. Instead, only local estimates $K_Z{}^{(n)}$ of the derivative can be obtained:

$$f'(Z''_n) \approx K_Z{}^{(n)} = \frac{f(Z''_n) - f(Z''_{n-1})}{Z''_n - Z''_{n-1}}.$$

Hence

$$K_Z{}^{(n)} = \frac{Z''_{n+1} - Z''_n}{Z''_n - Z''_{n-1}}.$$

It can be shown that

$$\alpha'' - Z''_{n+1} \approx -\frac{K_Z{}^{(n)}}{1 - K_Z{}^{(n)}}(Z''_{n+1} - Z''_n)$$

It follows that provided $K_Z{}^{(n)} \leqslant 0.5$

then $$|\alpha'' - Z''_{n+1}| \leqslant |Z''_{n+1} - Z''_n|$$

pointing to the adequacy of the Willmott (1964) method of ascertaining cyclic equilibrium in these cases. Where, however,

$$0.5 < K_Z{}^{(n)} < 1$$

we observe that

$$|\alpha'' - Z''_{n+1}| > |Z''_{n+1} - Z''_n|$$

and the "Aitken convergence criterion" should be used.

REPONSE TRANSITOIRE DES REGENERATEURS A ECOULEMENT PERIODIQUE

Résumé—Le stockage de chaleur, intrinsèquement lié au régénérateur, impose une inertie qui gouverne la réponse instantanée aux changements de conditions opératoires. On présente, sous forme graphique et adimensionnelle, les réponses d'un régénérateur thermique initialement en régime cyclique établi, à des changements échelons de la température d'entrée du gaz et du débit de gaz. On montre que l'inertie thermique est caractérisée par une longueur réduite Λ dans le cas équilibré. On étudie l'effet du déséquilibre sur l'inertie thermique, dans l'opération de régénération.

ÜBERGANGSFUNKTION PERIODISCH DURCHSTRÖMTER REGENERATOREN

Zusammenfassung—Die wärmespeichernde Packung, der Kernteil eines thermischen Regenerators, besitzt eine Trägheit, welche eine augenblickliche Anpassung an veränderte Betriebsbedingungen verhindert. In dimensionsloser, graphischer Form werden die Übergangsfunktionen eines thermischen Regenerators, der sich ursprünglich in zyklischem Gleichgewicht befindet, für den Fall permanenter, schrittweiser Aenderungen der Gaseintrittstemperatur und des Gasmengenstromes angegeben. Es wird gezeigt, daß die thermische Trägheit im Gleichgewichtsfall über die reduzierte Länge Λ erfaßt werden kann. Der Einfluß des Nichtgleichgewichtes in den Regeneratorbetriebsbedingungen auf die thermische Trägheit wird untersucht.

ПЕРЕХОДНЫЕ ХАРАКТЕРИСТИКИ ТЕПЛОВЫХ РЕГЕНЕРАТОРОВ ПЕРИОДИЧЕСКОГО ДЕЙСТВИЯ

Аннотация — Теплоизолирующая прокладка, применяемая в тепловом регенераторе, вызывает инерцию, которая исключает мгновенный отклик на изменение рабочих условий. В статье в безразмерном графическом виде представлены зависимости характеристик теплового регенератора, первоначально находящегося в циклическом равновесии, от постоянных ступенчатых изменений температуры газа на входе и расхода газа. Показано, что приведенная длина Λ параметризует тепловую инерцию в случае равновесия. Исследуется эффект дисбаланса в работе теплового регенератора за счет тепловой инерции.